EXERCICE№9

1) $\lim\limits_{x \to 1^+} \left(\frac{-2}{x-1}\right) = -\infty$; $\lim\limits_{x \to 1^-} \frac{-2}{x-1} = +\infty$

2)

x	$-\infty$	-1		1	$+\infty$
x^2-1	+	0	-	0	+

$\lim\limits_{x \to (-1)^+} \left(\frac{-2x^3}{x^2-1}\right) = -\infty$ (de la forme $\frac{2}{0^-}$) ; $\lim\limits_{x \to (-1)^-} \left(\frac{-2x^3}{x^2-1}\right) = +\infty$

3)

x	$-\infty$	1		2	$+\infty$
x^2-3x+2	+	0	-	0	+

$\lim\limits_{x \to 1^-} \left(\frac{-3}{x^2-3x+2}\right) = -\infty$; $\lim\limits_{x \to 1^+} \left(\frac{-3}{x^2-3x+2}\right) = +\infty$

4) pour $x>0$; $\sqrt{1+x^2} = \sqrt{x^2(1+\frac{1}{x^2})} = x\sqrt{1+\frac{1}{x^2}}$

Pour $x<0$; $\sqrt{1+x^2} = -x\sqrt{1+\frac{1}{x^2}}$ d'où $\lim\limits_{x \to +\infty} \frac{\sqrt{1+x^2}}{x} = \lim\limits_{x \to +\infty} \sqrt{1+\frac{1}{x^2}} = 1$

et $\lim\limits_{x \to -\infty} \frac{\sqrt{1+x^2}}{x} = \lim\limits_{x \to -\infty} -\sqrt{1+\frac{1}{x^2}} = -1$

EXERCICE№10

1) Pour $x \neq -1$: $f(x) = \frac{(x+1)(x^2+2x-4)}{(x+1)} = x^2+2x-4$

$\lim\limits_{x \to (-1)} f(x) = \lim\limits_{x \to (-1)} (x^2+2x-4) = -5$

$g(x) = \begin{cases} f(x) & \text{si } x \neq -1 \\ -5 & \text{si } x = -1 \end{cases}$ est le prolongement par continuité de f en (-1)

2) $f(x) = \frac{1-\sqrt{2}\cos x}{4\left(x-\frac{\pi}{4}\right)}$; on pose $X = x - \frac{\pi}{4}$ alors $\begin{matrix} x \to \frac{\pi}{4} \\ X \to 0 \end{matrix}$

$\lim\limits_{x \to \frac{\pi}{4}} f(x) = \lim\limits_{X \to 0} \frac{1-\sqrt{2}\cos(X+\frac{\pi}{4})}{4X} = \lim\limits_{X \to 0} \frac{1-\sqrt{2}(\cos\frac{\pi}{4}.\cos X - \sin\frac{\pi}{4}.\sin X)}{4X}$

$= \lim\limits_{X \to 0} \frac{1-\cos X+\sin X}{4X} = \lim\limits_{X \to 0} \frac{1}{4}\left[\underbrace{\frac{1-\cos X}{X}}_{\to 0} + \underbrace{\frac{\sin X}{x}}_{\to 1}\right] = \frac{1}{4}$

$g(x) = \begin{cases} f(x) & \text{si } x \neq \frac{\pi}{4} \\ \frac{1}{4} & \text{si } x = \frac{\pi}{4} \end{cases}$

3) $f(x) = \frac{16(\cos(4x)-1)}{(4x)^2}$; on pose $X = 4x$

$\lim\limits_{X \to 0} 16\left[\frac{\cos X-1}{X^2}\right] = \lim\limits_{X \to 0} \left[-16\left(\frac{1-\cos X}{X^2}\right)\right] = -8$ d'où : $g(x) = \begin{cases} f(x) & \text{si } x \neq 0 \\ -8 & \text{si } x = 0 \end{cases}$

Qarray.tn - Aqra Blech - Qarray.tn